Complementary cosmological tests of RSII brane models

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In this paper we explore observational bounds on flat and non-flat cosmological models in Type II Randall-Sundrum (RSII) branes. In a first analysis, we consider current measurements of the expansion rate $H(z)$ (with two priors on the local Hubble parameter) and 288 Type Ia supernovae from the Sloan Digital Sky Survey (within the framework of the mlcs2k2 light-curve fitting method). We find that the joint analysis involving these data is an interesting tool to impose limits on the brane tension density parameter (Ω_λ) and that the spatial curvature has a negligible influence on Ω_λ estimates. In order to obtain stronger bounds for the contribution of the Ω_λ we also add in our analysis the baryon oscillation peak (BAO) and cosmic microwave background radiation (CMB) observations by using the so-called CMB/BAO ratio. From this analysis we find that the Ω_λ contribution is less than 4.10^{-5} (1σ).

I. INTRODUCTION

Since the formulation of Kaluza-Klein theory in the beginning of last century, concerning the unification of gravity and electromagnetism in a five-dimensional space, the study of extra dimensions have attracted much attention in Physics. Nowadays there are great interest and much speculation about the existence of extra dimensions and their influence over our four-dimensional world, motivated mainly by the braneworld scenarios, in which our four-dimensional spacetime is viewed as a submanifold isometrically embedded in a space of higher dimensions[1, 2, 3, 4, 5, 6].

A relevant aspect of the braneworld scenarios is that matter and all fields are confined to the brane (below a certain energy level which is expected to be of Tev order) and only gravity can propagate into the bulk [1, 2, 3, 4]. Because of this characteristic, extra dimensions might have large scale as compared to the tiny Planck scale, assumed by the Kaluza-Klein theory, without any conflict with the observations so far. In particular, the RSII model admits the existence of a hidden extra dimension which has an infinity length [4]. If the hidden extra dimensions are large, in principle, they can be detected more easily by exploring the signature they might leave in our four-dimensional space. This could be done by trying to detect deviations of four-dimensional laws in domains where gravity was not empirically tested yet. Pursuing this goal, some research teams have investigated the validity of the inverse square law of gravity at sub-millimeter scales recently [7, 8, 9]. According to them, there is no evidence of extra dimensions down to a length scale of about $50\mu m$ [9]. In cosmology, there are also many studies about the impact of extra dimensions in the cosmic evolution considering distinct braneworld models[6, 10, 11]. Here we are interested in discuss some cosmological tests of RSII model.

The RSII model is distinguished from other braneworld models because it is the first one which, surpassing the paradigm of compact extra dimensions, shows the possibility that we live embedded in a space with an extra dimension of infinite size. Despite the fact that gravity can have access to the infinite extra dimension, a negative cosmological constant $\Lambda_{(5)}$ defined in the bulk ensures the existence of a massless graviton mode, which is confined to the brane, and the suppression of the massive modes. These two characteristics together, on their turn, guarantee that the Newtonian law is recovered at distances much larger than the curvature scale $\ell = \sqrt{-6/\Lambda_{(5)}}$ established by the five-dimensional cosmological constant[4].

The laboratory tests of the inverse square law, that have found no trace of extra dimension, as we already mentioned, then suggest that the curvature scale ℓ could not be greater than $10^{-4}m$ [9]. With such a small value, it seems that the extra dimension will play no significant role in the recent phase of evolution of the Universe, which, according to Type Ia supernovae (SNe Ia) data and based on the LCDM model, is undergoing an accelerated expansion. Needless to say, however, that any theory should be tested in different scales and by means of all kind of physical phenomena

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as much as possible. Hence, in this paper, we intended to use a recent SNe Ia data combined with measurements of H(z) as an independent and new test for the RSII model in the cosmological scale.

As is well known, in the cosmological context, the differences between the RSII model and the standard cosmological model can be synthesized as modifications of the Friedmann equation. Admitting that the brane is a Roberton-Walker three-dimensional space, then the effective field equation of gravity induced on the brane yields the modified Friedmann equation [10]:

$$\left(\frac{H}{H_0}\right)^2 = \Omega_\Lambda + \Omega_m \left(\frac{\rho}{\rho_0}\right) + \Omega_\kappa \left(\frac{a_0}{a}\right)^2 + \Omega_\lambda \left(\frac{\rho}{\rho_0}\right)^2 + \Omega_u \left(\frac{a_0}{a}\right)^4 \tag{1}$$

where a is the scale factor, H is the Hubble parameter, ρ is the density of the dust matter confined to the brane. The number zero as a subscript indicates the current value of the quantities. The parameters Ω_Λ, Ω_m and Ω_κ are the density parameters of the four-dimensional cosmological constant $\Lambda_{(4)}$, of the matter and of the spatial curvature, respectively. They have the usual definition $\Omega_\Lambda = \Lambda_{(4)}/3H_0^2$, $\Omega_m = 8\pi G\rho_0/3H_0^2$ and $\Omega_\kappa = -\kappa/a_0^2 H_0^2$. The new terms are: the density parameter of the positive brane tension (λ)

$$\Omega_\lambda = \frac{48\pi G \rho_0^2}{H_0^2 \lambda}$$

and the density parameter of the so-called dark radiation $\Omega_u = m/a_0^4 H_0^2$, where m is another free constant of the model.

It is important to emphasize that all terms of Eq. (1) are quantities that inhabit the brane, except the dark radiation, which is essentially a five-dimensional quantity and, therefore, represents a direct influence of the bulk geometry over the brane [6, 10]. The physical origin of the dark radiation can be understood by examining this picture from the perspective of the embedding formalism. It can be shown that a homogeneous and isotropic three-dimensional space, i.e., a Robertson-Walker 3-space, can be isometrically embedded into the AdS$_5$-Shwarzschild spacetime [12, 13], which is a black hole solution in five dimensions with a negative cosmological constant. From this point of view, the expansion of the observable Universe is a consequence of the motion of the brane in the static AdS$_5$-Scwharzschild space. Moreover, it can also be shown that the trajectory of the brane is dictated by equation (1), where the scale factor localizes the brane in the bulk and m is the mass of the bulk black hole [12, 13].

Other remarkable difference of the equation (1), in comparison with the usual Friedmann equation, is that it depends on the energy density squared. The coefficient of this quadratic term is the parameter density Ω_λ which is inversely proportional to the brane tension. So, the higher the brane tension the lower the effects of the extra dimension in our world. This happens because, in the RSII model, the brane tension is connected to the five dimensional cosmological constant $\Lambda_{(5)}$. A high tension corresponds to a high cosmological constant and, therefore, a strong suppression of the massive graviton modes. Of course measurements of the brane tension λ by estimates of Ω_λ would give us important information about the influence of extra dimension in the cosmic evolution.

At this point, it is interesting to highlight that ρ^2 arises too in cosmological models based on other theories as the Loop Quantum Cosmology [14] and non-Riemannian theories [15]. Therefore, although the RSII brane model is the main motivation of our discussion, the constraints on the density parameter Ω_λ we find here could also be used to put bounds on the parameters of those theories.

The modified Friedmann equation depends on the five density parameters that should be determined by empirical data. The first confrontation of RSII model with SNe Ia data was done in Ref. [16]. The authors found, by using the Perlmutter samples [17], that SNe Ia data provide weak constraints on the brane tension. The simplest case with $m = 0$ and with only dust matter on the brane yields $\Omega_k = -0.9$, $\Omega_\lambda = 0.04$, $\Omega_m = 0.59$ and $\Omega_\Lambda = 1.27$ as the best fit according to the χ^2 method. Considering these numbers as an unrealistic result, the prior $\Omega_m = 0.3$ was added in the flat model. In this case, $\Omega_\lambda < 0.037$ at 2σ level, by using the Perlmutter sample A [16].

In Ref. [18], another data set of SNe Ia was employed in order to constrain the parameters of the RSII model. The best fit, in the flat model with the prior $\Omega_u = 0 \pm 0.1$, is $\Omega_\lambda = 0.026$, $\Omega_m = 0.15$ and $\Omega_\Lambda = 0.80$, by using data of the Riess gold samples [19]. Once again, the constraints were not so strong. For instance, $\Omega_m \in [0, 0.62]$ at 2σ level. The curved model was also investigated, however, in this case, it was necessary to impose some priors on density matter ($\Omega_m \simeq 0.3$) in order to get reasonable values for the density parameters.

The contribution of the ρ^2 in cosmology was again considered in Ref. [20], this time by using a data set of SNe Ia containing samples from [19, 21, 22]. In that paper, the authors also considered other tests like CMB and BAO in order to constrain the density parameter of the RSII model. However, taking into account only the SNe Ia data, they found similar results. Indeed, without imposing any additional prior, the best fit obtained ($\Omega_k = 0.34$, $\Omega_m = 0.0$, $\Omega_\lambda = 0.044$ and $\Omega_\Lambda = 0.646$) was again not very reasonable.

Therefore, based on these previous results, it seems that, as SNe Ia data do not constrain strongly the density parameter of the matter, we do not find realistic results without assuming some priors for the matter density. In this

paper, we want to show that this can be avoid if we consider a joint analysis with SNe Ia data and $H(z)$ measurements. We explore flat and non-flat cosmologies. As we shall see, with only 21 data points, $H(z)$ test gives supplementary information which naturally put realistic bounds on the parameters without manipulating priors to the matter density.

For the sake of simplicity, we are going to admit hereafter that $m = 0$. Of course, this case corresponds to the simplest model, in which there is no black hole in the original bulk and, therefore, no dark radiation in the brane. Nevertheless, we have to mention that there are some papers that have focused their attention on the analysis of dark radiation in RSII branes also by using SNe Ia data, see for example[23].

In order to obtain stronger bounds for the parameters of the RSII model, we have also considered the CMB and BAO cosmological tests. However, differently from [20], our analysis is based on the CMB/BAO ratio which is expected to be weakly cosmological model dependent according to [24].

The paper is organized as follows. In section II, we give a short description of the observational data we have used. The corresponding constraints on the cosmological parameters are investigated in section III. The article is ended with a summary of the the main results in the conclusion section.

II.   DATA SETS

A.   SNe Ia Sample

In our analyses we have considered the SNe Ia data sample from the Sloan digital sky survey [24, 25]. This sample is a combination of 103 SNe Ia with redshifts $0.04 < z < 0.42$, discovered during the first season of the Sloan Digital Sky Survey-II (SDSS-II), with a comprehensive and consistent reanalysis of other datasets [19, 26, 27], resulting in a combined sample of 288 SNe Ia. These 103 data filled the redshift desert between low- and high-redshift SNe Ia of the previous SNe Ia surveys. Kessler et al. (2009) used two light-curve fitters to obtain the SNe Ia distance moduli, namely, MLCS2K2 [28] and SALT2 [29]. In this paper we use the SNe Ia distance moduli obtained via MLCS2K2 calibration since this method does not have cosmological model dependence [30]. Furthermore, as is largely employed in literatura, all the results in our analysis (see next section) from SNe Ia data are derived by marginalizing the likelihood function over the nuisance parameter [19, 31].

B.   H(z) measurements

In recent years $H(z)$ measurements of the Hubble parameter have been used to constrain cosmological parameters [32, 33, 34, 35, 36, 37]. The idea underlying this approach is based on the measurement of the differential age evolution of the massive and passively evolving early-type galaxies as a function of redshift, which provides a direct estimate of the Hubble parameter $H(z) = -1/(1+z)dz/dt \approx -1/(1+z)\Delta z/\Delta t$. It is important to stress that direct measurements of $H(z)$ at different redshifts is also possible through measurements of the radial component of baryonic acoustic oscillations [38] (for recent $H(z)$ reviews see [33]).

In this paper, we have used the 21 $H(z)$ measurements [34, 39, 40] in the redshift range $0 < z < 1.75$. The H_0 influence on our results is explored by considering two priors in analysis: 68 ± 2.8 km/s/Mpc, from a median statistics analysis of 553 measurements of H_0 [35], and 73.8 ± 2.4 km/s/Mpc, the most recent one based on HST measurements [41]. Following [37] we have assumed that the distribution of H_0 is a Gaussian with one standard deviation width σ_{H_0} and mean $\bar{H}_0$, so that we can build the posterior likelihood function $\mathcal{L}_H(\mathbf{p})$ that depends only on the parameters $\mathbf{p}$ by integrating the product of $\exp(-\chi_H^2/2)$ and the H_0 prior likelihood function $\exp[-(H_0 - \bar{H}_0)^2/(2\sigma_{H_0}^2)]$,

$$\mathcal{L}_H(\mathbf{p}) = \frac{1}{\sqrt{2\pi\sigma_{H_0}^2}} \int_0^\infty e^{-\chi_H^2(H_0,\mathbf{p})/2} e^{-(H_0-\bar{H}_0)^2/(2\sigma_{H_0}^2)} dH_0. \tag{2}$$

In this way, to obtain our results from $H(z)$ measurements we maximize the likelihood $\mathcal{L}_H(\mathbf{p})$, or equivalently minimize $\chi_H^2(\mathbf{p}) = -2\ln\mathcal{L}_H(\mathbf{p})$, with respect to the parameters $\mathbf{p}$ to find the best-fit parameter values to flat and non-flat universes (see next section).

C.   Ratio CMB/BAO

As it is largely known, the CMB constrains from the so-called shift parameter R [42] and the BAO measurement A [43] have been commonly used to constrain non-standard models, but this approach is not completely correct since
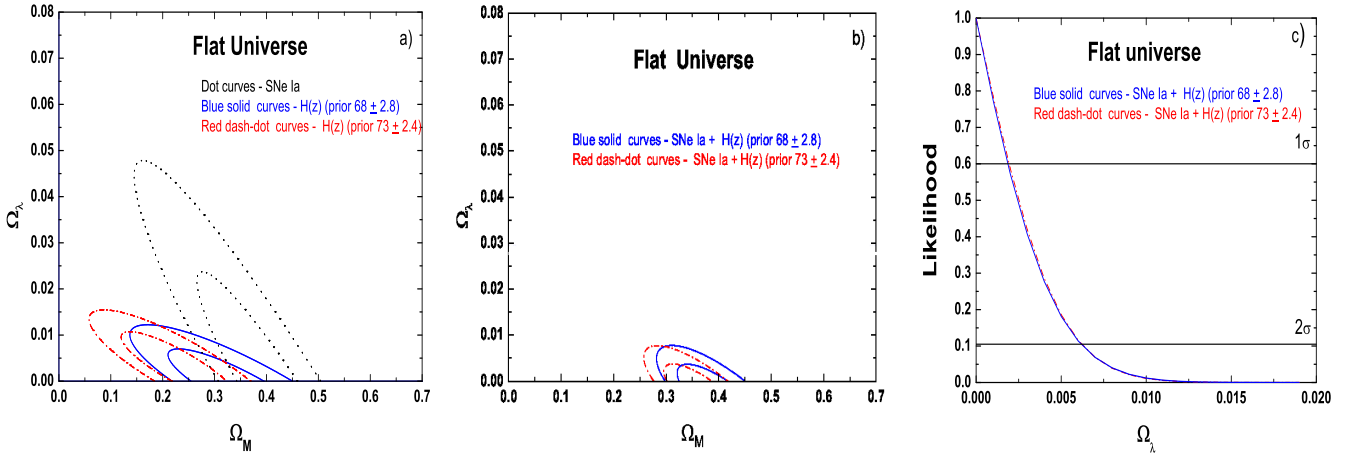
FIG. 1: Confidence contours on the plane $\Omega_\lambda - \Omega_M$ to flat Universe. Fig. 1a) Contours are drawn to 1σ and 2σ. The black dots corresponds to bounds from SNe Ia. Through this paper the H(z) priors 68 ± 2.8 and 73.8 ± 2.4 correspond to blue solid and red dash-dot curves, respectively. Fig. 1b) shows the contours to 1σ and 2σ from our joint analysis. Fig. 1c) displays the likelihoods to Ω_λ.

these quantities are derived by using parameters close to standard wCDM [24, 42, 44]. In order to avoid some bias in our results we follow Ref.[24] and use the ratio CMB/BAO. This product cancels out some of the dependence on the sound horizon size at last scattering and a more model-independent constraint can be achieved in statistical analysis. In our analysis we use two ratio CMB/BAO measurements in redshifts $z = 0.2$ and 0.35, namely [24] (with one standard deviation error bars)

$$\frac{d_A(z_*)}{D_V(0.2)} = 17.55 \pm 0.65, \qquad (3)$$

$$\frac{d_A(z_*)}{D_V(0.35)} = 10.10 \pm 0.38 \qquad (4)$$

where $d_A(z_*)$ is the comoving angular-diameter distance to recombination, z_* is the redshift of the last-scattering surface ($z_* = 1090$) and $D_V(z)$ is the so-called dilation scale. As emphasized in Ref. [24] these two poins have a correlation coefficient of 0.337 [45]. In this point, we have to assume that brane models do not change the physics of the pre-recombination epochs [20].

III. RESULTS

A. Flat Universe

The results of our statistical analyses to flat universe are shown in Figures 1a, 1b and 1c. Figure 1a shows contours of 1σ and 2σ on the $\Omega_\lambda - \Omega_M$ plane when SDSS (MLCS2K2) and H(z) data points are considered separately (through this paper the priors 68 ± 2.8 and 73.8 ± 2.4 correspond to blue solid and red dash-dot curves, respectively). In this case Ω_Λ can be find by using simply $\Omega_\Lambda = 1 - \Omega_\lambda - \Omega_M$. By considering two free parameters, we can find from the SNe Ia sample (1σ): $\Omega_M = 0.37 \pm 0.11$ and $\Omega_\lambda = 0.0 + 0.022$ ($\chi^2 = 237.84$). On the other hand, from H(z) data points we find: $\Omega_M = 0.32 \pm 0.1$ and $\Omega_\lambda = 0.0 + 0.006$ ($\chi^2 = 15.87$), and $\Omega_M = 0.24 \pm 0.12$ and $\Omega_\lambda = 0.0 + 0.01$ ($\chi^2 = 16.9$) when the priors 68 ± 2.8 km/s/Mpc and 73.8 ± 2.4 km/s/Mpc are used on H_0, respectively. Note that no significant conflict between the results from $H(z)$ priors is found and the bounds on Ω_λ from 21 H(z) data points are tighter than ones from 288 SNe Ia. Moreover, the contours from SNe Ia and H(z) analyses are almost ortogonal each other and a joint analysis can be used to impose tighter limits on the space parameters.

Figure 1b shows our results from a joint analysis. We find $\Omega_M = 0.37 \pm 0.05$ and $\Omega_\lambda = 0.0 + 0.0033$ ($\chi^2 = 255.45$) and $\Omega_M = 0.33 \pm 0.04$ and $\Omega_\lambda = 0.0 + 0.0031$ ($\chi^2 = 260.45$) by using SNe Ia + H(z) with the priors 68 ± 2.8 km/s/Mpc and 73.8 ± 2.4 km/s/Mpc on H_0, respectively.
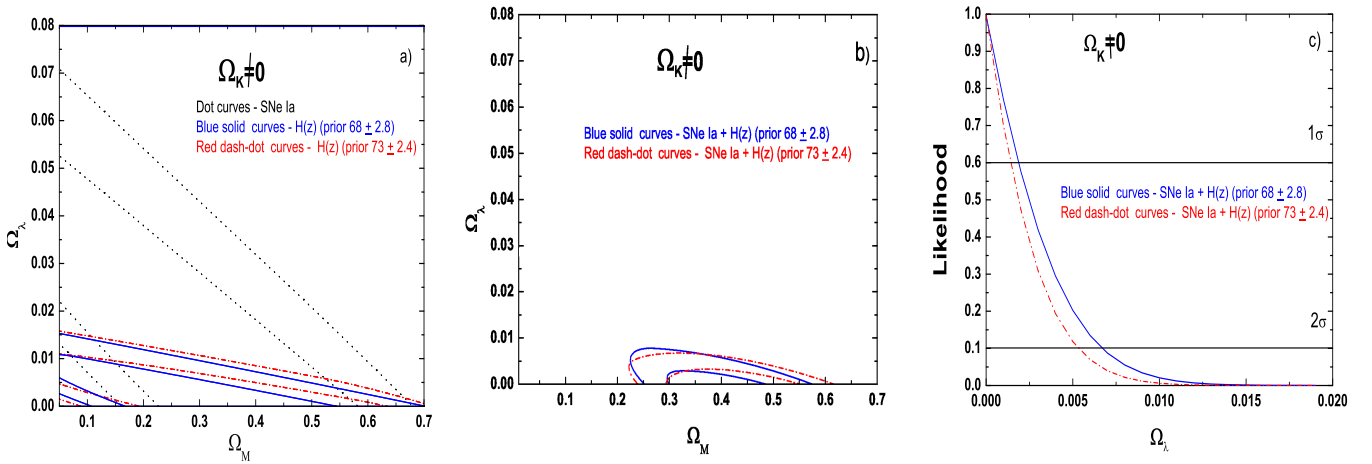
FIG. 2: Confidence contours on the plane $\Omega_\lambda - \Omega_M$ to flat universe. Fig. 2a) Contours are drawn to 1σ and 2σ. The black dots corresponds to bounds from SNe Ia. Through this paper the H(z) priors 68 ± 2.8 and 73.8 ± 2.4 correspond to blue solid and red dash-dot curves, respectively. Fig. 2b) shows the contours to 1σ and 2σ from our joint analysis. Fig. 2c) displays the likelihoods to Ω_λ.

Panel 1c display the likelihood for the Ω_λ parameter, marginalizing on Ω_M. In this case we obtain $\Omega_\lambda = 0.0 + 0.0018$ to both priors.

B. Non-Flat Universe

In this section, we allow deviations from flatness to prove the robustness of the previous Ω_λ constraints using SNe Ia and H(z) data points. The results of our statistical analyses are shown in figures 2a, 2b and 2c (here we marginalize over Ω_Λ). Figure 2a show contours of 1σ and 2σ on the $\Omega_\lambda - \Omega_M$ plane when SDSS (MLCS2K2) and H(z) data points are considered separately. In this case we can not constrain Ω_M and Ω_λ simultaneously and, for instance: $\Omega_M = 0.50$ and $\Omega_\lambda = 0.005$, and $\Omega_M = 0.20$ and $\Omega_\lambda = 0.035$ are permitted with high degree of confidence by using the SNe Ia sample. Furthermore, $\Omega_\lambda = 0$ is excluded at 2σ to $\Omega_M = 0.1$ and it is permitted at 1σ to $\Omega_M = 0.3$. Similar conclusion can be done to results from H(z). According with these comments a joint analysis is necessary in order to break the degenerescency on the $(\Omega_\lambda, \Omega_M)$ plane.

Figure 2b displays the results by using a joint analysis involving SNe Ia and H(z) data points. We find (1σ): $\Omega_M = 0.38 \pm 0.09$, $\Omega_\lambda = 0.0 + 0.0012$ ($\chi^2 = 255.33$) and $\Omega_M = 0.41 \pm 0.08$, $\Omega_\lambda = 0.0 + 0.001$ ($\chi^2 = 260.21$) with the priors 68 ± 2.8 km/s/Mpc and 73.8 ± 2.4 km/s/Mpc on H_0, respectively. On the other hand, by marginalizing on Ω_M we find: $\Omega_\Lambda = 0.65 \pm 0.17$ and $\Omega_\Lambda = 0.80 \pm 0.18$, respectively.

Panel 1c display the likelihood for the Ω_λ parameter, marginalizing on Ω_M and Ω_Λ. In this case we obtain (1σ) $\Omega_\lambda = 0.0 + 0.0018$ and $\Omega_\lambda = 0.0 + 0.0016$ to 68 ± 2.8 km/s/Mpc and 73.8 ± 2.4 km/s/Mpc H_0 priors, respectively. Therefore, the limits on Ω_λ are equivalent to the flat case and we see that the geometry has a negligible influence on estimates for this combination of data.

Finally, in order to obtain tighter limits on space parameters we add in our analyses two points of the so-called ratio CMB/BAO. In figures 3a, 3b and 3c we show our results. We find (at 1σ): $\Omega_M = 0.28 \pm 0.03$, $\Omega_\lambda = 0.0 + 0.0001$ ($\chi^2 = 262.88$) and $\Omega_M = 0.28 \pm 0.03$, $\Omega_\lambda = 0.0 + 0.00006$ ($\chi^2 = 264.21$) with the priors 68 ± 2.8 and 73.8 ± 2.4, respectively. By marginalizing on Ω_M we find: $\Omega_\Lambda = 0.65 \pm 0.10$ and $\Omega_\Lambda = 0.68 \pm 0.10$ for each case, respectively. It is very important to stress that the limits derived by using SNe Ia + H(z) + CMB/BAO are more cosmological model independent than previous analyses where the A and R quantities were used [20]. The likelihood curves are plotted in figure 3c. We obtain $\Omega_\lambda = 0.0 + 0.00004$ and $\Omega_\lambda = 0.0 + 0.00002$ (1σ) to 68 ± 2.8 and 73.8 ± 2.4 priors, respectively.

IV. CONCLUSIONS

In this paper, we have studied observational bounds on cosmologies based on the flat and non-flat RSII brane models. Previous papers showed that SNe Ia data do not provide strong constraints on the parameters of RSII model. Moreover, the best fits did not give realistic values unless some priors to the matter and curvature densities
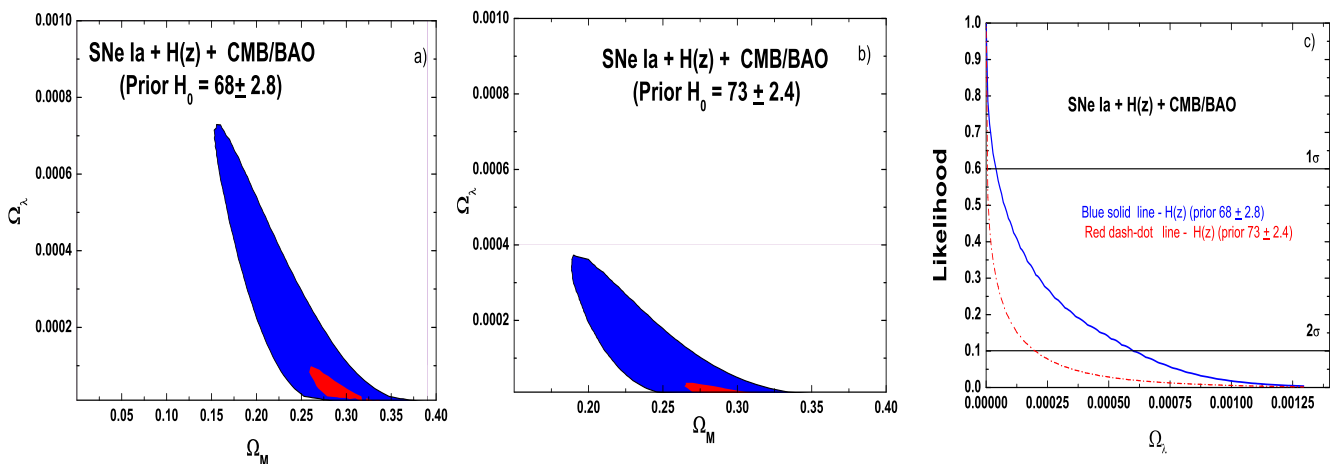
FIG. 3: Confidence contours on the plane $\Omega_\lambda - \Omega_M$ to non-flat universe by using SNe Ia $+ H(z) +$ BAO/CMB ratio. Fig. 2a) Contours are drawn to 1σ and 2σ, H_0 prior corresponds to 68 ± 2.8 km/s/Mpc. Fig. 2b) Contours are drawn to 1σ and 2σ, H_0 prior corresponds to 73.8 ± 2.4 km/s/Mpc. Fig. 3c) displays the likelihoods to Ω_λ.

were assumed. However, as we have shown here, when we consider the joint analysis of 288 SNe Ia (SDSS by using MLCS2K2 method) and 21 H(z) measurements we obtain consistent results with other observations [47] for matter density without imposing such priors. Moreover, the constraints on Ω_λ are weakly dependent on the curvature parameter and H_0 priors (68 ± 2.8 and 73.8 ± 2.4 km/s/Mpc), we have obtained at 1σ: $\Omega_\lambda \approx 0.0 + 0.002$ to flat and non-flat Universes (see Figs. 1c and 2c). It is important to emphasize that in the near future it will be possible to determine around 1000 $H(z)$ values of the Hubble parameter at a 15% accuracy level [32, 48]. This fact shows that upcoming $H(z)$ measurements plus SNe Ia may become a useful tool to impose constraints on Ω_λ.

We also have added the so-called CMB/BAO ratio in our analysis in order to obtain tighter limits on Ω_λ. As is known, this ratio is more model independent than analyses done by using the A and R quantities which have parameters close to standard wCDM in their derivations. The joint analysis (SNe Ia+H(z)+CMB/BAO) gives the following values at 1σ: $\Omega_\lambda = 0.0 + 0.00004$ and $\Omega = 0.0 + 0.00002$ with the priors 68 ± 2.8 and 73.8 ± 2.4, respectively. Finally, it is important to stress that the constraints for the brane tension found here are much weaker than the upper limit put by the analysis of Big Bang Nucleosynthesis (BBN) $\left(\Omega_\lambda < 10^{-20}\right)$ [20, 49], however, our analysis is based on different physical phenomena and on physical processes that happened at a different cosmological epoch as compared to BBN and, therefore, constitute an independent test that must be performed in order to check the consistency of the model at large scale.

V. ACKNOWLEDGMENTS

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